Exponential stability for a structure with interfacial slip and frictional damping

Carlos A. Raposo *

There are few manuscripts that deal with systems of interfacial slip, we cite [1, 3] and the recent work [2] where was established the existence of smooth finite dimensional global attractors for the corresponding solution semigroup. In [1], Hansen and Spies derived the mathematical model (1) for two-layered beams with structural damping due to the interfacial slip

$$\begin{aligned} &\rho u_{tt} + G(\psi - u_x)_x = 0,\ x \in (0,1),\ t \geq 0, \\ &I_\rho(3S_{tt} - \psi_{tt}) - G(\psi - u_x) - D(3S_{xx} - \psi_{xx}) = 0,\ x \in (0,1),\ t \geq 0, \\ &3I_\rho S_{tt} + 3G(\psi - u_x) + 4\delta_0 S + 4\gamma_0 S_t - 3DS_{xx} = 0,\ x \in (0,1),\ t \geq 0, \end{aligned} \tag{1}$$

where $u(x,t)$ denotes the transverse displacement, $\psi(x,t)$ represents the rotation angle, and $S(x,t)$ is proportional to the amount of slip along the interface at time t and longitudinal spatial variable x. The coefficients $\rho, G, I_\rho, D, \delta_0, \gamma_0$ are the density, the shear stiffness, mass moment of inertia, flexural rigidity, adhesive stiffness, and adhesive damping of the beams. The equation $3I_\rho S_{tt} + 3G(\psi - u_x) + 4\delta_0 S + 4\gamma_0 S_t - 3DS_{xx} = 0$ describes the dynamics of the slip. In [4] was proved that the frictional damping $4\gamma_0 S_t$ created by the interfacial slip alone is not enough to stabilize the system (1) exponentially to its equilibrium state. The natural question is: does the dissipation process caused by the full damped system imply the exponentially stability? In this work we prove the exponential stability for a laminated beam consisting of two identical layers of uniform density, which is a system closely related to the Timoshenko beam theory, taking into account that an adhesive of small thickness is bonding the two layers and produce the interfacial slip. It is assumed that the thickness of the adhesive bonding the two layers is small enough so that the contribution of its mass to the kinetic energy of the entire beam may be ignored.

References

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*Department of Mathematics, Federal University of São João del-Rei, mail to: raposo@ufsj.edu.br