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INFLUENCE OF SAMPLE RATE AND DISCRETIZATION METHODS IN THE IDENTIFICATION OF SYSTEMS WITH HYSTERESIS

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Abstract: This paper shows how the sample rate and discretization methods affect the parameter identification of a NARX model, when applied to systems with hysteresis, whose is a model non-linear behavior usually found in electromagnetic devices. A Bouc-Wen model for a magneto-rheological damper is used as a system to be identified by a NARX model, considering the above mentioned scenario and a least-square based technique is used in this work to estimate model parameters.

keywords: Analysis and Control of Nonlinear Dynamical Systems with Practical Applications; Modeling, Numerical Simulation and Optimization; Nonlinear dynamics and Complex Systems; Systems with Hysteresis.

1. INTRODUCTION

System identification is a procedure widely used to obtain mathematical models from a system input and output data. Once the model is identified, it is possible to obtain an estimate of system's static and dynamic behavior and this estimate can be used as an auxiliary tool for several applications, such as model-based control [6].

Mathematical models are used in systems project and control, simulation and training. Since simulations always deal with discrete-time systems and models, it is useful to study how the sample rate and discretization methods affect a model performance.

Besides, system identification can be also used to characterize and control systems with hysteresis. Hysteresis is a hard to model non-linear behavior usually found in actuators, motors, sensors and others electromagnetic devices involving memory effects between input and output [5, 8].

Concerning nonlinear systems with hysteresis, Bouc-Wen model is frequently used for modelling, as in the identification of magneto-rhelogical dampers [7]. This model consists, essentially, in a first-order nonlinear differential equation that relates the input displacement to the output restoring force in a hysteretic way. However, a correct estimation of parameters must be well performed. Otherwise, the model may become unstable [7]. Furthermore, the use of the Bouc-Wen model in a control scenario is a very hard task, since the inverse model is not easily obtained [2, 11].

Given the limitations presented by the model above mentioned, the use of polynomial NARX (Nonlinear autoregressive with exogenous inputs) models has become an alternative for modeling systems with hysteresis. Given the model structure, the inverse model can be used in a feedforward control. Martins and Aguirre [2] showed that polynomial NARX model can be used to identify the non-linearities of a Bouc-Wen model. However, the mentioned work did not investigated how the sample rate and the discretization method may affect the model during parameter estimation.

It is well known that sample rate affects structure selection of NARX model [10]. However, to the best of our knowledge, there is no research showing how the sample rate or discretization method affect the parameter identification of a NARX model, when applied to systems with hysteresis. This is the main aim of this work.

This paper is organized as follows: a brief introduction and bibliographical review has been presented in this section. Section 2 shows some basics concepts, necessary for understanding the whole text. The methods used to identified the influence of sample rate and discretization methods are presented in section 3. The results as well as the discussion are presented in section 4, while concluding remarks and perspectives for future research are shown in section 5.

2. PRELIMINARIES

2.1. NARX polynomial models

A NARX model ((Non-linear AutoRegressive model with eXogenous inputs) can be represented as [14]:

$$\begin{aligned} y(k) &= F^{\ell}[y(k-1), ..., yk(-n_y) \\ &\quad u(k-d), ..., u(k-n_u)] + e(k), \end{aligned} \tag{1}$$

where $u(k)$ and $y(k)$ are respectively the input and output signals, $e(k)$ accounts for uncertainties and possible noise and F^{ℓ} is a polinomial with degree ℓ. This polynomial model has been extensively studied and used to model nonlinear systems. However, it only has been recently applied to systems with hysteresis [11].

2.2. Discretization Methods

Euler's method, which employs the idea that a tangent line can be used to approaching the values of a function in a small neighborhood of the point tangency [12]. Thereby if you wish approximations $y_1, y_2, ..., y_m$ for exact results $y(x_1), y(x_2), ..., y(x_m)$. if unknown the value $y(x_1)$, the approximation is y_1. For this, draw the tangency T on the curve $y(x)$ in point $(x_0, y(x_0)$:

$$y(x) - y(x_0) = (x - x_0)y(x_0) \tag{2}$$

If $x = x_1$ and remember $y(x_0) = y_0$, $x_1 - x_0 = h$, $y'(x_0) = f(x_0), y(x_0))$ and $y_1 = y(x_1)$, we get:

$$y_1 = y_0 + hf(x_0, y(x_0)) \tag{3}$$

To estimate y_2, we may advance the index i:

$$y_i + 1 = y_i + hf(x_i, y_i), i = 0, 1, ..., m-1 \tag{4}$$

therefore:

$$f(x_i, y_i) = (y_i + 1 - y_i)/h \tag{5}$$

The Eq. 5 offer an estimate for the output of a differential model, which h means the integration step. An improvement of this estimative may be given by the Euler's improved method [12]:

$$y_i + 1 = y_i + hf(x_n/n, y_n/n), x_n = x_1 + x_2 ... x_n \tag{6}$$

3. METHODS

Initially a Bouc-Wen model for a magneto-rheological damper [7] was considered:

$$\begin{aligned} f &= c_1\dot{\rho} + k_1(x - x_0), \\ \dot{\rho} &= (1/c_0 + c_1)[\alpha z + c_0\dot{x} + k_0(x - \rho)], \\ \dot{z} &= -\gamma|\dot{x} - \dot{\rho}|z|z|^{n-1} - \beta(\dot{x} - \dot{\rho})|z|^n + A(\dot{x} - \dot{\rho}), \\ \alpha &= \alpha_a + \alpha_b u_{bw}, \\ c_1 &= c_{1a} + c_{1b}u_{bw}, \\ c_0 &= c_{0a} + c_{0b}u_{bw}, \\ \dot{u}_{bw} &= -\eta(u_{bw} - E). \end{aligned} \tag{7}$$

where f represents the restoring force, c_1 and c_0 are the intensity of the damping dynamic, E is the input voltage, the variable x is the displacement and $\dot{x}$ is the velocity of the model.

Using fourth order Runge-Kutta with $h = 0,002s$, the Bouc-Wen model was identified by a NARX model in [2]:

$$\begin{aligned} y(k) &= 0.8347y(k-1) + 0.442u_3(k-1) \\ &\quad +0.6704u_2(k-1) \times u_1(k-1) \\ &\quad -0.4648u_3(k-1) \times u_2(k-1) \times y(k-1). \end{aligned} \tag{8}$$

where u_1 is the voltage (E), u_2 reffers to the velocity $(\dot{x})$ and u_3 =sign(v).

To verify how discretization methods and sample rate may interfere in parameter estimation, the model 7 was integrated by mean of Euler's and Euler's improved methods, considering different values of integration step, considering the same identification procedure proposed by [2].

To make sure that the Euler and Euler improved discretization methods were applied correctly, the values of voltage (E) and the displacement (x) were set equal to $E(t) = 0.5sin(2\pi 0.5t) + 1.6V$ and $x(t) = sin(2\pi 3t)cm$, with an integration step of 0.00002s and 0.0002s to Euler and Euler improved respectively.

Then, in order to obtain the NARX parameters, the inputs were considered as two independent realizations of uniformly distributed random numbers. As the hysteresis is a quasi-static behaviour [9], it was used a low-pass filter with a cutoff frequency of 6 Hz to filter the inputs. The filter is a FIR Blackman-Harris windowed, used to attenuate undesired frequency components.

We obtained two set of data for each method of discretization, one for estimating the parameters and the other for model validation in both cases. Besides that, to verify the influence of the sampling rate in the estimation of parameters a decimation of the data was applied using a decimation factor D = 2 and obtaining new parameters.

Model performance was quantified by the normalized root mean square error, described as:

$$RMSE = \frac{\sqrt{\sum_{k=1}^{N}(y(k)-\hat{y}(k))^2}}{\sqrt{\sum_{k=1}^{N}(y(k)-\bar{y}(k))^2}}, \tag{9}$$

being $\hat{y}(k)$ the model output, $\bar{y}(k)$ the mean of the measured output $y(k)$.

4. RESULTS AND DISCUSSION

Least square technique was applied to obtain the model parameters. The table below present the parameters obtained using data from different methods of discretization for the model structure presented in Eq. (8).

Table 1 – Estimated parameters.

	Euler	Euler Improved	Runge-kutta
$\theta 1$	0.910097160	0.813860860	0.84313718185
$\theta 2$	0.0134750630	0.0204311070	0.04358176662
$\theta 3$	0.106085470	0.0658137510	0.68215468661
$\theta 4$	-0.503710160	-0.630839010	-0.470401060

Similarly, it was obtained using the model parameters decimated data, with a decimation factor equal to 2. The values are presented in the following table:

Table 2 – Estimated parameters after decimation of data.

	Euler	Euler Improved	Runge-kutta
$\theta 1$	1.271140	0.526529950	0.84094462508
$\theta 2$	0.00657382870	0.0142511560	0.04434415683
$\theta 3$	0.136723070	0.0938125270	0.69031130036
$\theta 4$	-0.859561440	-0.444743290	-0.4755978577

Analyzing tables 1 and 2, it is possible to realize that is a clear influence in the methods of discretization and the sample rate in the parameters. The Euler discretization method showed the worst method among all the analyzed, because it presented the biggest RMSE, as provided by Table 3. Furthermore, it is important to mention that there are also cases where the RMSE tends to ∞, meaning that the model output diverges. Figures 1, 2 and 3 show the force curve normalized with and without decimation data obtained in relation to samples, considering $h = 0.002s$.

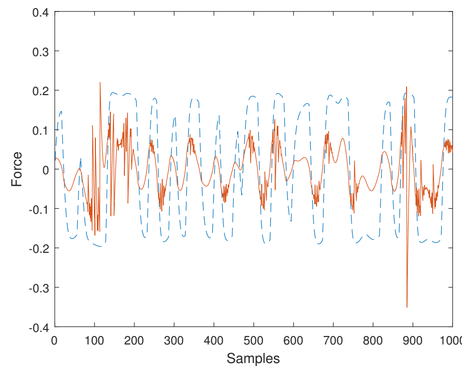

Figure 1 – Validation of the model obtained through the Euler method for discretization of the Bouc-Wen model, with $h = 0.002s$. (-)Bouc-Wen model,(–) identified model

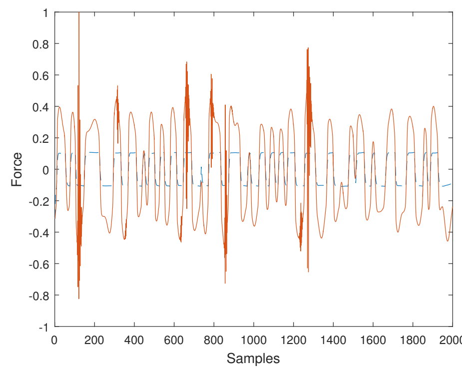

Figure 2 – Validation of the model data obtained by the Improved Euler method for discretization of the Bouc-Wen model, with $h = 0.002s$. (-)Bouc-Wen model,(–) identified model

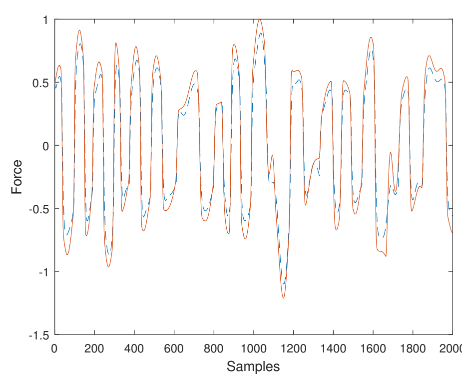

Figure 3 – Model validation obtained through data of Runge-Kutta method for discretization of the Bouc-Wen model, with $h = 0.002s$. (-)Bouc-Wen model,(–) identified model

The influence of the sampling rate in the estimation of the parameters is verified in Table 3. For each method, the first value corresponds to the model performance identified using $h = 0.002$s for Runge Kutta, $h = 0.0002$s for Improved Euler and $h = 0.00002$s for Euler, and the second one corresponds to model performance after decimation of data.

Table 3 – e_{rms} computed for first and decimated validation data sets

Euler		Improved Euler		Runge-Kutta	
e_{rms} 1	e_{rms} 2	e_{rms}1	e_{rms} 2	e_{rms}1	e_{rms}2
1.859	∞	0.704	0.801	0.184	0.314

5. CONCLUSION

This work presented a analysis of the influence of the discretization method and the sample rate the estimation of the parameters of a system with hysteresis through NARX polynomial models. To this end, data were obtained applying the Euler, Euler improved with random noise and Runge-Kutta methods to a Bouc-Wen model. After that, the parameters were estimated and compared among themselves.

The results obtained presented that the discretization method used to obtain data from one system influence the estimation of parameters the same, the parameter estimation using data obtained from the Euler's method was the most ineffective, with the largest mean square error.

Besides that, it can be seen, also, the influence of the sampling rate in the estimation of parameters. Applying a decimation factor, the model obtained by Euler method was not able to represent the hysteresis of the system, diverging completely.

Future work should present other discretization methods and verify how these obtained model can be used in a model based and hysterese compensation control scenario.

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