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A GENERALIZED LINEAR MATRIX INEQUALITY APPROACH TO MULTIOBJECTIVE SYSTEMS IDENTIFICATION

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Abstract. *This paper presents a generalization of the technique presented in Silva et al. (2012), a linear matrix inequality approach to multiobjective parameter estimation in system identification. It is shown how to incorporate in the model parameters any kind of auxiliary information that can be expressed as affine information. Furthermore, it is investigated the performance of the generalized LMI technique, incorporating several system's characteristics in the model. Finally, it is investigated the relevance of the incorporation of different types of auxiliary information in the model parameters, i.e., which ones are most important and really necessary to obtaining representative models.*

Keywords: *Control Applications, Modeling and Control, Identification*

1 Introduction

System Identification is a very relevant branch of science which studies different ways to model and analyse systems, attempting to find patterns in observations (Billings, 1980). To identify a system, it is necessary to propose a model which is able to describe its several characteristics, considering dynamic, gain and static behaviour.

A model can be defined as the set of hypotheses about structure or behaviour of a physical system. Under a mathematical point of view, a model is an abstraction of a real system expressed by mean of equations.

In engineering, multiobjective system identification techniques can be applied in the identification of many systems such as in the modelling of electric heaters, DC-DC buck converters, chemical systems (Eklund et al., 2007) and in other areas such as in biological systems (Buonomo et al., 2013) and economic (Griffith, 1992).

Literature is replete with system identification techniques, using several mathematical and computational representations (Wang et al., 2012; Farina e Piroddi, 2010). Among them, it can be cited Neural Networks, Fuzzy Logic, NARX Models (Non-linear AutoRegressive model with eXogenous Input) and Individuals Based Models. Polynomial NARX representation allows, with relative ease, the incorporation of a priori system information in the model. Such information (static curve, fixed points location and static gain, and other - see Nepomuceno et al. (2007) for a list of them) may not be embedded into dynamical data. Thus, the addition of such information can significantly improve the model quality regarding its robustness and system representativeness.

From works of Johansen (1996), systems identification specialists began to investigate the possibility of using auxiliary information of the system in the modelling scenario. Then, a new approach for system identification comes up, the so-called multiobjective system identification (Nepomuceno et al., 2007; Martins et al., 2013).

In parallel, Linear Matrix Inequalities (LMI) techniques have been evolving exponentially, mainly used in the development of robust control techniques, in which uncertainties regarding the model should be considered (Araujo et al., 2015; Lan e Zhou, 2011). LMI theory is pretty recent and even more recent is its application as system identification techniques (Estrada-Manzo et al., 2016; Wang et al., 2012). Hiramoto (2012) presents a method which uses the Schur complement to obtain LMIs conditions equivalent to the problem of estimating model parameters via least squares methods.

On the other hand, Silva et al. (2012) showed how to use, in addition to dynamic data, information about the location of fixed points in a bi-objective optimization problem for identification of a chaotic system. However, the use of a generic number of auxiliary information in obtaining model parameters is not well defined so far. Moreover, there is not a study about which information, among that available previously, are really relevant in obtaining the model parameters, leading to a global and representative model.

The purpose of this paper is to develop a methodology for multiobjective system identification with LMI, considering a generic number of objectives, in order to incorporate any information expressed as affine information (Nepomuceno et al., 2007), a generalization of the technique presented by Silva et al. (2012). Furthermore, the relevance of three affine information is investigated, i.e., which information is really important and necessary to be used in the modelling procedure, in order to obtain good models. The use of LMI to incorporate static curve and static gain in the model parameters was not investigated yet, being this another aim of the present work.

2 Preliminaries

21 System identification

Obtaining a model from experimental data requires five basic steps Ljung (1987): i) Data acquisition; ii) Choice of the mathematical representation; iii) Structure detection; iv) Parameter estimation and v) Model validation.

In order to acquire data with dynamic information embedded, a persistently exciting signal has to be used as an input. In this sense, a Pseudo-Random Binary Signal (PRBS) was used as a dynamic input. After acquired, the data were split in two distinct groups, one of them was used for identification and the other one for validation.

The technique developed on this work is applicable to any classes of non-linear models which are linear in the parameters. So, a polynomial NARX representation will be used, as an example of such linear representation parameters. Since the structure selection Baldacchino et al. (2012) is not the main purpose of this work, the Error Reduction Ratio (ERR) was applied with the Akaike Information Criterion (AIC), in order to obtain the model structure. Model parameters was obtained using the generalized LMI multiobjective approach, presented in this work in Section 3.

Once the models are already obtained, it is necessary to validate them. Thus, it can quantified which ones are really representative, considering different system features. In this work, some indexes were used to validate the obtained models. Since information about dynamic behaviour, static curve and gain will be incorporated in the models, three indexes are used for model validation: RMSE (Root Mean Squared Error), SMSE (Static curve Mean Square Error) and GMSE (Gain Mean Square Error).

The normalized RMSE index is given by:

$$\mathrm{RMSE} = \frac{\sqrt{\sum_{k=1}^{N}[y(k) - \hat{y}(k)]^2}}{\sqrt{\sum_{k=1}^{N}[y(k) - \bar{y}]^2}}, \tag{1}$$

where $\hat{y}(k)$ is the infinity step-ahead model simulation and $\bar{y}$ is the average value of the measure $y(k)$. This index measures the error in a unit of measurement consistent with the real dynamic data Nepomuceno et al. (2007). Good dynamic models are those ones which have normalized RMSE lesser than unity. This means that, on average, the squared error given by the model is lesser than the mean squared error given by the average of the time series.

The SMSE and GMSE indexes can be expressed as:

$$\mathrm{(S)(G)MSE} = \frac{\sum_{k=1}^{N_{SG}} \left(y_{SG} - \hat{y}_{SG}\right)^2}{N_{SG}} \tag{2}$$

where N_{SG} is the number of points used for validation (static and gain), y_{SG} is the real value of the affine information and $\hat{y}_{SG}$ its value estimated by model.

22 *NARX* polynomial models

NARX models Billings (1980) describe non-linear systems by mean of difference equations which are linear in the parameters, relating the current output with past combinations of outputs and inputs. NARX models can be used in control problems where the goal is to find a simple description for the system. In particular, the NARX polynomial model can be represented as:

$$y(k) \;=\; F^{\ell}[y(k-1), \cdots, y(k-n_y), u(k-1), \cdots u(k-n_u)] + \Xi(k),$$

where $y(k)$ is the output and $u(k)$ is the exogenous input at time k. $\Xi(k)$ is the prediction error. n_y and n_u are the maximum lags considered for the output $y(k)$ and input $u(k)$. The function F^{ℓ} can represent a wide variety of functions, including linear and non-linear functions. In this paper, F^{ℓ} is restricted to non-linear polynomial functions.

23 Linear Matrix Inequality (LMI)

Linear matrix inequalities have been used as an important tool, especially in the analysis and control of uncertain systems. The improvement of techniques for the solution of convex optimization problems makes LMIs techniques more useful in several practical applications. Problems which are composed by LMIs, are smart and have a simple and easy computing solution. Mathematically, every LMI can be written as:

$$F(x) = F_0 + \sum_{i=1}^{L} x_i F_i \succ 0 \tag{3}$$

where x_i is the i−th decision variables. L is the total number of decision variables. $F_i \in \Re^{n\times x}$ are known symmetric matrices.

A mathematical tool in the analysis and construction of LMI conditions is the Schur complement. Schur complement is especially useful in the conversion of a set of non-linear matrix inequality in linear matrix inequality (LMI).

Schur complement: Let $Q(x)$ and $R(x)$ be two symmetric matrices ($Q(x) = Q^T(x)$ and $R(x) = R^T(x)$) that depend on the decision variable x, and $S(x)$ be another matrix. Both LMI below are equivalent:

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} \succ 0, \tag{4}$$

given:

$$\begin{aligned} R(x) &\succ 0 \\ Q(x) - S(x)R(x)^{-1}S^T(x) &\succ 0. \end{aligned} \tag{5}$$

■

24 DC-DC Buck Converter.

A power electronic system known as DC-DC buck converter was used to verify the presented approach. DC-DC buck converter (Fig. 1), is a step down converter which consists on the voltage regulation on the load. The duty cicle of the converter can be controlled by a MOSFET or IGBT. In this work a MOSFET IRF840 was used. Duty cycle is defined as the ratio where the converter is on during the total operation time. The PRBS (Pseudo-Random Binary Signal) was used as an input signal, applied to the switch. This signal is persistently exciting, in such a way that the system can present in the output its non-linear dynamics.

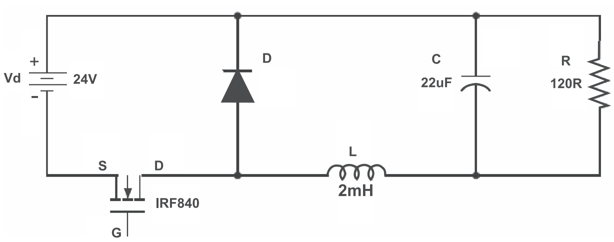

Figure 1- DC-DC buck converter.

The static behavior of the DC-DC buck converter is described by the following equation:

$$\bar{y} = \frac{4}{3}v_d - \frac{v_d}{3}\bar{u} \tag{6}$$

where $v_d = 24V$ is the DC voltage source and $\bar{u}$ is the static input. Since it is a step down converter, the gain is a negative value (actually, an attenuation). For the DC-DC buck converter used, specifically, the attenuation is constant and can be given by: $\frac{d\bar{y}}{d\bar{u}} = -\frac{v_d}{3} = -8V$

3 Methodology

This section presents the generalized LMI approach to multiobjective system identification, used to estimated the parameters of a polynomial NARX model. This method allows the inclusion of any system feature that may be written by the model as affine information (Nepomuceno et al., 2007).

Every affine information can be expressed as $y_i = \hat{y}_i + \xi_i$ or $y_i = \Psi_i\hat{\theta} + \xi_i$, where y_i is the i−th affine information (fixed points, static curve, static gain, dynamic data and other system features), $\hat{y}_i$ is the estimated value, given by the multiplication between the matrix of regressors (Ψ_i) and the estimated parameter vector ($\hat{\theta}$). ξ_i is an error vector. In a least square approach, the problem is to minimize the following cost function (Hiramoto, 2012):

$$J_{LS} = (y - \hat{y})^T (y - \hat{y}) = \left(y - \Psi\hat{\theta}\right)^T \left(y - \Psi\hat{\theta}\right). \quad (7)$$

It is known, by definition, that $J_{LS} > 0$, since this is a quadratic function. It is also desired that the function has its value as small as possible, say, lesser than λ, determined below. Thus, according to Hiramoto (2012), Equation 7 can be rewritten as:

$$\begin{aligned} (y - \hat{y})^T I (y - \hat{y}) &\prec \lambda I \\ (y - \hat{y})^T I (y - \hat{y}) - \lambda I &\prec 0, \\ \lambda I - (y - \hat{y})^T I^{-1} (y - \hat{y}) &\succ 0. \end{aligned} \quad (8)$$

where I is the identity matrix. Using the Schur complement, and since the matrix $I \succ 0$, the above LMIs can be represented by the LMI presented in Equation 9 Hiramoto (2012):

$$\begin{bmatrix} \lambda I & (y - \hat{y}) \\ (y - \hat{y})^T & I \end{bmatrix} \succ 0. \quad (9)$$

Therefore the optimization problem can be defined: minimize λ subject to LMI represented by inequality 9, where $\hat{y} = \Psi\hat{\theta}$.

A cost function composed of two objectives, fixed points and dynamic data, proposed by Silva et al. (2012), can be expressed as:

$$J_C = \left[\omega_1\Psi_1^T(y_1 - \Psi_1\hat{\theta}) + \omega_2\Psi_2^T(y_2 - \Psi_2\hat{\theta})\right]^T \left[\omega_1\Psi_1^T(y_1 - \Psi_1\hat{\theta}) + \omega_2\Psi_2^T(y_2 - \Psi_2\hat{\theta})\right]$$

where the subscript 1 is related to dynamic data and 2 is related to fixed point. In Equation 10 is presented a weighted sum of squared error, expressing both information considered in the modelling process as affine information. Equation 10 can be rewritten as:

$$\begin{aligned} J_C &= \left[\sum_{i=1}^{2}\omega_i\Psi_i^T(y_i - \Psi_i\hat{\theta})\right]^T \left[\sum_{i=1}^{2}\omega_i\Psi_i^T(y_i - \Psi_i\hat{\theta})\right] \\ &= \left[\sum_{i=1}^{2}\omega_i\Psi_i^T y_i - \sum_{i=1}^{2}\omega_i\Psi_i^T\Psi_i\hat{\theta})\right]^T \left[\sum_{i=1}^{2}\omega_i\Psi_i^T y_i - \sum_{i=1}^{2}\omega_i\Psi_i^T\Psi_i\hat{\theta}\right] \end{aligned} \quad (10)$$

Equation 10 can be expanded to incorporate in the model parameters a generic number of system feature (say, Z informations), which can be written as affine information Nepomuceno et al. (2007). Therefore, rewriting it for the incorporation of Z objectives (system characteristics):

$$\begin{aligned} J_C &= \left[\sum_{i=1}^{Z} \omega_i \Psi_i^T (y_i - \Psi_i \hat{\theta})\right]^T \left[\sum_{i=1}^{Z} \omega_i \Psi_i^T (y_i - \Psi_i \hat{\theta})\right] \\ &= \left[\sum_{i=1}^{Z} \omega_i \Psi_i^T y_i - \sum_{i=1}^{Z} \omega_i \Psi_i^T \Psi_i \hat{\theta}\right]^T \left[\sum_{i=1}^{Z} \omega_i \Psi_i^T y_i - \sum_{i=1}^{Z} \omega_i \Psi_i^T \Psi_i \hat{\theta}\right] \end{aligned} \tag{11}$$

or:

$$J_C = \left[Y - \psi\hat{\theta}\right]^T \left[Y - \psi\hat{\theta}\right], \tag{12}$$

where: $Y = \sum_{i=1}^{Z} \omega_i \Psi_i^T y_i$ e $\psi = \sum_{i=1}^{Z} \omega_i \Psi_i^T \Psi_i$.

Due to the similarities between Equation 12 and Equation 7, the LMI condition can be written as the mono-objective problem with Z objectives incorporated into parameter estimation, as:

$$\begin{bmatrix} \lambda I & (Y - \psi\hat{\theta}) \\ (Y - \psi\hat{\theta})^T & I \end{bmatrix} \succ 0, \tag{13}$$

Therefore, the generalized multiobjective problem is to minimize the sum of squared error of information estimation ($\lambda \succ 0$), subject to the condition imposed by the LMI presented in inequality 13. Then, varying the values of ω_i (such that $\sum_{i=1}^{Z} \omega_i = 1$) different solutions (Pareto-set solutions) of the multiobjective optimization problem are obtained. Thus, the relevance and the importance of including different features system can be analysed.

4 Results and discussion

The approach developed in Section 3 was applied to obtain good models for a DC-DC buck converter. First of all, to obtain models in different regions, the set of weights associated with dynamic information, static information and gain (ω_1, ω_2 e ω_3, respectively) has been defined and are shown in Table 1. That is, a set of 15 weights (15 models) was chosen to represent different points of the set of available solutions. Among all, it was also considered the mono-objective case, that is, a condition in which two of the weights assume null values. It should be emphasized that the condition $\omega_1 = 1$, $\omega_{2,3} = 0$ is analogous to the least squares method. In the least square method only the dynamic data are used to obtain the model parameters.

Table 2 lists all the models obtained by the sets of weights shown in Table 1, and also the stability of each one. About the model structure, the Akaike information criterion (AIC) suggested the use of 7 regressors in the model, which have been selected through the Error Reduction Ratio (ERR) and are showed at the top of Table 2.

Table 1- Weights assigned to dynamic (ω_1), static (ω_2) and gain (ω_3) objectives.

Model	ω_1	ω_2	ω_3
$\mathcal{M}_1$	0.2	0.4	0.4
$\mathcal{M}_2$	0.4	0.2	0.4
$\mathcal{M}_3$	0.4	0.4	0.2
$\mathcal{M}_4$	0.6	0.2	0.2
$\mathcal{M}_5$	0.8	0.2	0
$\mathcal{M}_6$	0.8	0	0.2
$\mathcal{M}_7$	1	0	0
$\mathcal{M}_8$	0.2	0.6	0.2
$\mathcal{M}_9$	0.2	0.8	0
$\mathcal{M}_{10}$	0	0.8	0.2
$\mathcal{M}_{11}$	0	1	0
$\mathcal{M}_{12}$	0.2	0.2	0.6
$\mathcal{M}_{13}$	0	0.2	0.8
$\mathcal{M}_{14}$	0.2	0	0.8
$\mathcal{M}_{15}$	0	0	1

The models $\mathcal{M}_{10}$, $\mathcal{M}_{11}$, $\mathcal{M}_{13}$ and $\mathcal{M}_{15}$ were unstable. This can be explained because these models do not use any dynamic information in the parameter estimation procedure. In front of this, becomes evident the need and the importance of inclusion of dynamic data in the parameter estimation, since their inclusion is directly related to the models stability.

The RMSE, SMSE and GMSE indexes validate the models obtained considering dynamic data (RMSE), static curve (SMSE) and static gain (GMSE) and are shown in Table 3. The indexes were not calculated for dynamically instable models.

The model $\mathcal{M}_7$ ($\omega_1 = 1$ $\omega_2 = 0$ and $\omega_3 = 0$) had the best dynamic performance, the model $\mathcal{M}_9$ ($\omega_1 = 0,2$ $\omega_2 = 0,8$ and $\omega_3 = 0$) had the smallest static error and the model $\mathcal{M}_{14}$ ($\omega_1 = 0,2$ $\omega_2 = 0$ and $\omega_3 = 0,8$) had the smallest static gain. It is also observed that the model $\mathcal{M}_9$ had a good dynamic behaviour, with RMSE less than one, but not as good as the model $\mathcal{M}_7$. This suggests that a small loss in dynamic capability (decreasing ω_1) can substantially improve other model capabilities.

The dynamic behaviour (V), the static curve (y_s) and the static gain (G) of the models $\mathcal{M}_7$, $\mathcal{M}_9$, and $\mathcal{M}_{14}$ are shown in Figures 2 and 3, respectively. The models $\mathcal{M}_7$ and $\mathcal{M}_9$ have satisfactory dynamic behaviour. This can not be observed in the model $\mathcal{M}_{14}$, once it converges to a fixed value. The model $\mathcal{M}_7$ has satisfactory static performance only in the range $1 \leq \bar{u} \leq 4$, the same range of the dynamic input data (PRBS input signal covers from 2 V to 2.5 V). Moreover, the same model was not able to precisely estimate the gain in a wide range of operation points. Finally, the model $\mathcal{M}_9$ has a good performance, considering aspects of static curve and dynamic behaviour. This shows the relevance of incorporating the static curve information to obtaining more global models, valid in a wide region of operation points.

In addition, it is necessary to highlight that the decision of which model is *better*, or which model has to be used is not easy. For this reason, the choice should consider the specific application of the model. If a model with good approximation with a set of three dynamic, static curve and gain is required, it is suggested to choose $\omega_i = 1/Z$, $\forall i = 1, \cdots, Z$, with Z being the number of objectives. If this combination of weights leads to an unstable model, close values of the weights can be used (always observing the constraint $\sum_{i=1}^{Z} \omega_i = 1$).

Table 2- Structure and parameters obtained by mean of the multiobjective LMI approach presented. In the first column, the capital letter S stands for Stable and the letter U stands for Unstable.

Mod.	$y(k-1)$	$y(k-2)$	$y(k-1)^2$	$u(k-1)^2$	$y(k-2)^2$	$y(k-2)y(k-1)$	C
$\mathcal{M}_1$ (S)	0.0018	0.9590	0.0737	-0.0357	-0.0433	-0.0301	0.7051
$\mathcal{M}_2$ (S)	0.0070	0.9100	0.0702	-0.0698	-0.0438	-0.0259	1.4912
$\mathcal{M}_3$ (S)	0.0055	0.8634	0.0680	-0.1241	-0.0425	-0.0238	2.2061
$\mathcal{M}_4$ (S)	0.0104	0.7974	0.0635	-0.1731	-0.0429	-0.0187	3.2566
$\mathcal{M}_5$ (S)	0.0023	0.1428	0.0275	-0.8564	-0.0331	0.0190	13.7588
$\mathcal{M}_6$ (S)	0.0063	0.7249	0.0481	-0.1843	-0.0559	-0.0019	6.4044
$\mathcal{M}_7$ (S)	0.0001	-0.2873	0.0194	-0.5637	-0.0362	0.0522	13.9648
$\mathcal{M}_8$ (S)	0.0016	0.9291	0.0721	-0.0669	-0.0427	-0.0285	1.1664
$\mathcal{M}_9$ (S)	0.0002	0.1433	0.0279	-0.8569	-0.0329	0.0184	13.7326
$\mathcal{M}_{10}$ (U)	0	1	1.7414	0	-0.8707	-0.8707	0
$\mathcal{M}_{11}$ (U)	0	-0.0640	0.0010	-0.0640	0	0	1.0236
$\mathcal{M}_{12}$ (S)	0.0020	0.9665	0.0738	-0.0255	-0.0437	-0.0300	0.6259
$\mathcal{M}_{13}$ (U)	0	1	100.8441	0	-50.4221	-50.4221	0
$\mathcal{M}_{14}$ (S)	0.0008	0.9748	0.0561	-0.0151	-0.0627	-0.0136	4.0815
$\mathcal{M}_{15}$ (U)	0	8.5E6	22.6E6	0	22.7E6	22.6E6	0

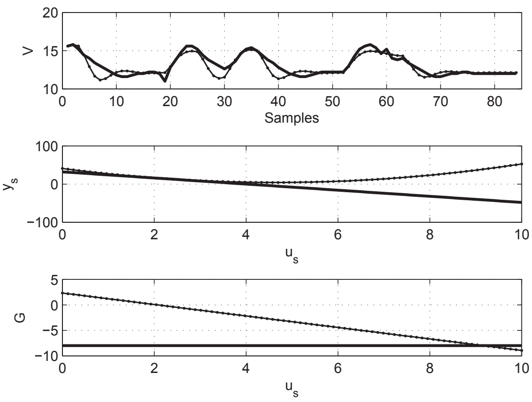

Figure 2- Model ($\mathcal{M}_7$), the best dynamic performance of simulated models. (—) real values and ($-\bullet$) values estimated by the model.

Furthermore, the LMI approach allows the imposition of parametric constraints in the model parameters, only adding others LMI with this information ($\hat{\theta}_1 \succ 0$, $\hat{\theta}_2 + \hat{\theta}_3 \succ 1$, for instance). If the resulting problem is feasible and convex, the solution can be easily computationally obtained.

5 Conclusion

In this paper a new generalized multiobjective LMI method for parameter estimation was proposed to estimate the parameter of linear in parameters models such as NARX models. The

Table 3- Indexes used to quantify the models performance.

Model	RMSE	SMSE	GMSE
$\mathcal{M}_1$	1.7003	0.0107	0.04531
$\mathcal{M}_2$	1.2154	0.1050	0.1676
$\mathcal{M}_3$	0.8072	0.0332	0.5616
$\mathcal{M}_4$	0.6324	0.1972	1.0595
$\mathcal{M}_5$	0.5783	0.0031	27.9621
$\mathcal{M}_6$	0.9093	58.1658	1.2481
$\mathcal{M}_7$	0.4867	1611	32.9019
$\mathcal{M}_8$	1.0724	0.0052	0.1660
$\mathcal{M}_9$	0.6040	0.0004	27.9730
$\mathcal{M}_{10}$	—	—	—
$\mathcal{M}_{11}$	—	—	—
$\mathcal{M}_{12}$	2.3484	0.0296	0.0222
$\mathcal{M}_{13}$	—	—	—
$\mathcal{M}_{14}$	1.0868	225.9607	0.0084
$\mathcal{M}_{15}$	—	—	—

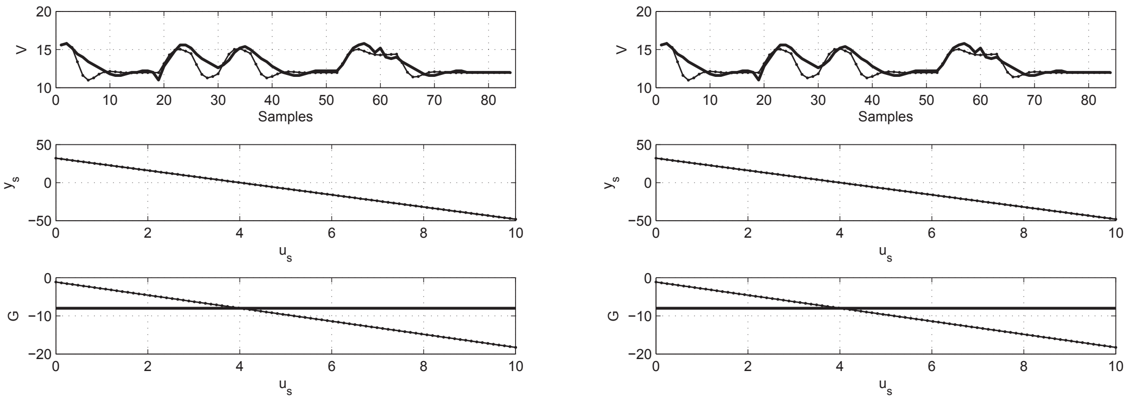

Figure 3- Model ($\mathcal{M}_9$ - left), the lowest static error of simulated models and model ($\mathcal{M}_{14}$ - rigth), the lowest gain error of simulated models. (—) real values and ($-\bullet$) values estimated by the model.

incorporation of Z affine information was considered. Furthermore, the importance of incorporating three kinds of affine information (dynamic, static curve and static gain) was analysed and discussed in the parameter estimation, their peculiarities, benefits and losses. It should be emphasized that the technique is useful to incorporate any other information that can be expressed as affine information.

The main advantages of these procedure is that it does not present numerical conditioning problems, once it is not required matrices inversion. The matrices inversion are often required in mono-objective and multiobjective optimization techniques (least squares-based approaches).

The use of LMI approach applied in systems identification is potentially relevant. As a future research, it is expected to use such tools to structure selection in non-linear models with parametric linearity.

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