SIMULATION OF CHUA’S CIRCUIT BY MEANS OF INTERVAL ANALYSIS

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Abstract: The Chua’s circuit is a paradigm for nonlinear scientific studies. It is usually simulated by means of numerical methods under IEEE 754-2008 standard. Although the error propagation problem is well known, little attention has been given to the relationship between this error and inequalities presented in Chua’s circuit model. Taking the average of round mode towards +∞ and −∞, we showed a qualitative change on the dynamics of Chua’s circuit.

Keywords: Bifurcation Analysis and Applications, Modelling, Chua’s Circuit, Numerical Simulation and Optimization, Analysis and Control of Nonlinear Dynamical Systems with Practical Applications.

1. INTRODUCTION

We live in a complex world [6] where the majority of dynamical systems are nonlinear. The choice of non-linear models brings with it an inevitable increase in the complexity [1]. One of the systems most widely used as a model to study the nonlinear dynamics and chaos is the Chua’s circuit [5].

The Chua’s circuit, developed by Leon O. Chua in 1984, is known to exhibit chaotic behaviour similar to the system proposed by Lorenz [2]. It is a simple electronic network which exhibits a variety of phenomena, such as strange attractors and bifurcations. The circuit consists of two capacitors, an inductor, a linear resistor, and a nonlinear resistor [5].

The numerical computation uses floating point arithmetic in most computers under the IEEE 7542-2008 standard [3, 4]. This standard, in some sense, is a systematic approximation of the real arithmetic and it is represented by a finite subset of the real numbers. As a result, some properties of the arithmetic of real numbers are not guaranteed for the floating-point [4]. So, it is necessary attention to the limitations of computers with respect to scientific computing.

Given the arithmetic floating point is an approximation of the real numbers, small errors are generated during the process and simulations [8]. These errors in the eyes of many users and are made subjectively considered unnoticeable, but can strongly influence the results.

This article focuses on the computational numerical round modes. The inequalities presented the Chua’s circuit model are addressed in parallel with interval analysis [7] of the circuit. Taking the average of round mode towards +∞ and −∞, we showed a qualitative change on the dynamics of Chua’s circuit.

2. OBJECTIVES

The purpose of this article is to show the influence of computer round modes applied to Chua’s circuit. Using interval arithmetic circuit in the simulation mode to a careful analysis of these results and those originating traditional computational rounding, that is, according to the rules of IEEE standard 754-2008 floating-point arithmetic. Since the circuit is chaotic behaviour, the influence of error propagation in the system dynamics is also the target of the study.

3. METHODS

The Chua’s circuit is one of the most used systems as an example to study the non-linear dynamics and chaos. The circuit shown in Figure 1a is composed of linear elements, except the Chua’s diode, which shows non-linear behaviour as shown in Figure 1b. This non-linear element can be implemented by operational amplifiers [1]. The Equations of the
circuit are 1, 2 and 3.

\[
C_1 \frac{dv_{c_1}}{dt} = \frac{v_{c_2} - v_{c_1}}{R} - i_d(v_{c_1}) \tag{1}
\]

\[
C_2 \frac{dv_{c_2}}{dt} = \frac{v_{c_1} - v_{c_2}}{R} - i_d \tag{2}
\]

\[
L \frac{di_L}{dt} = -v_{c_2} \tag{3}
\]

Table 1: Values of components and constants used in the simulations.

<table>
<thead>
<tr>
<th>Components</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_1)</td>
<td>10 nF</td>
</tr>
<tr>
<td>(C_2)</td>
<td>100 nF</td>
</tr>
<tr>
<td>(L)</td>
<td>19 mH</td>
</tr>
<tr>
<td>(R)</td>
<td>1980 (\Omega)</td>
</tr>
<tr>
<td>(m_0)</td>
<td>-0.37 mS</td>
</tr>
<tr>
<td>(m_1)</td>
<td>-0.68 mS</td>
</tr>
<tr>
<td>(B_p)</td>
<td>1.1 V</td>
</tr>
</tbody>
</table>

Since \(V_{c_1}\) is the voltage across the capacitor \(C_1\), \(V_{c_2}\) is the voltage across the capacitor \(C_2\), \(i_L\) is the current through the inductor and the current in the diode is [1]:

\[
i_d(v_{c_1}) = \begin{cases} 
    m_0v_{c_1} + B_p(m_0 - m_1) & \text{for } v_{c_1} < -B_p \\
    m_1v_{c_1} & \text{for } |v_{c_1}| \leq B_p \\
    m_0v_{c_1} + B_p(m_1 - m_0) & \text{for } v_{c_1} > B_p
\end{cases} \tag{4}
\]

![Figure 1: The Chua’s circuit [1].](image)

(a) the Chua’ circuit and (b) curve of Chua diode (current-voltage characteristic).

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\end{cases} \tag{4}
\]

The orbit of a chaotic dynamic system is sensitive to initial conditions and aperiodic [9]. Although, for the same set of parameters and initial conditions, Figure 2b shows a periodic result, which is not sensitivity to initial condition.

5. CONCLUSION

It is observed that even though infinitesimal errors along the simulation can compromise the result. Interval analysis were employed as the round mode was adapted to set the mid value of an interval composed by the round modes towards to \(-\infty\) and \(+\infty\). So, what it could be noticed is that
the system simulated by tradition RK4 and IEEE-754-2008 round mode standard, that is, round to the nearest, presents a strange attractor with a probably chaotic behaviour. On the contrary, the proposed algorithm furnishes a beautiful periodical result. Which one is the correct? We do not know for sure. The simplicity of a periodical solution and the reasons presented to use an average of the round methods, help us to believe that this is the correct solution. We intend to build Chua’s circuit following the steps of [5] and compare this simulated results with experimental results. Although, this procedure seems a way to give a final answer, we should keep in mind that there is no way to implement a circuit that matches perfectly the set of equations and parameters given in this paper. This observation makes our investigation yet more fascinating!

6. Bibliography


7. APPENDIX

7.1. Algorithm 1

```matlab
%Initial Conditions
clear all
system_dependent('setround',-Inf);
ym=[-0.7 0 0];
system_dependent('setround',Inf);
yp=[-0.7 0 0];
tf=0.075;
h=1e-6;
tspan = 0:h:tf;
N=length(tspan);
for k=1:N-1
    system_dependent('setround',-Inf);
    aux = ode4(@chua,tspan(k:k+1),
                ym(k,:),yp(k,:));
    ym(k+1,:)=aux(2,:);
    system_dependent('setround',Inf);
    aux = ode4(@chua,tspan(k:k+1),
                yp(k,:),ym(k,:));
    yp(k+1,:)=aux(2,:);
end

%Figures
figure(1)
plot(1:N,ym(:,1),1:N,yp(:,1),'k')
figure(2)
plot3(ym(:,1),ym(:,2),ym(:,3),'k')
view(-16,24);grid;
```

7.2. Algorithm 2

```matlab
function out = chua(t,in,in2)
    x = in(1); y = in(2); z = in(3);
    x2=in(1);y2=in(2);z2=in(2);
    L = 19.2*10^(-3);
    C1 = 10 *10^(-9);
    C2 = 100 *10^(-9);
    R = 1978.5;
    G = 1/R;
    m0=-0.37*10^(-3);
    m1=-0.68*10^(-3);
    Bp=1.1;
    \Average +Inf and -Inf
    x=(x+x2)/2;
    y=(y+y2)/2;
    z=(z+z2)/2;
    \Diode
    if x >Bp
        g=m0*x+Bp*(m1-m0);
    elseif (x >= -Bp)&(x <= Bp)
        g=m1*x;
    else
        g=m0*x+Bp*(m0-m1);
    end
    \Chua's Circuit Equations
    xdot = (1/C1)*(G*(y-x)-g);
    ydot = (1/C2)*(G*(x-y)+z);
    zdot = -(1/L)*y;
    out = [xdot ydot zdot];
```