Complex Networks

Master of Science in Electrical Engineering

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5.2 Power-Law Degree Distributions

- We have seen that power laws are very particular distribution functions that appear in many situations.
- The most important property of a degree distribution is that it has to be normalisable.

$$1 = p_0 + \sum_{k=1}^{\infty} p_k = p_0 + c \sum_{k=1}^{\infty} k^{-\gamma} \tag{1}$$

where $p_0 = N_0/N$ is the fraction of isolated nodes in the network.

- Let $p_0 = 0$.

$$p_k = \frac{k^{-\gamma}}{\sum_{k=1}^{\infty} k^{-\gamma}} = \frac{k^{-\gamma}}{\xi(\gamma)} \quad \gamma > 1 \tag{2}$$

  where $c = 1/\xi(\gamma)$.

- Identifying a power-law behaviour and extracting the exponent γ can be a very tricky issue.

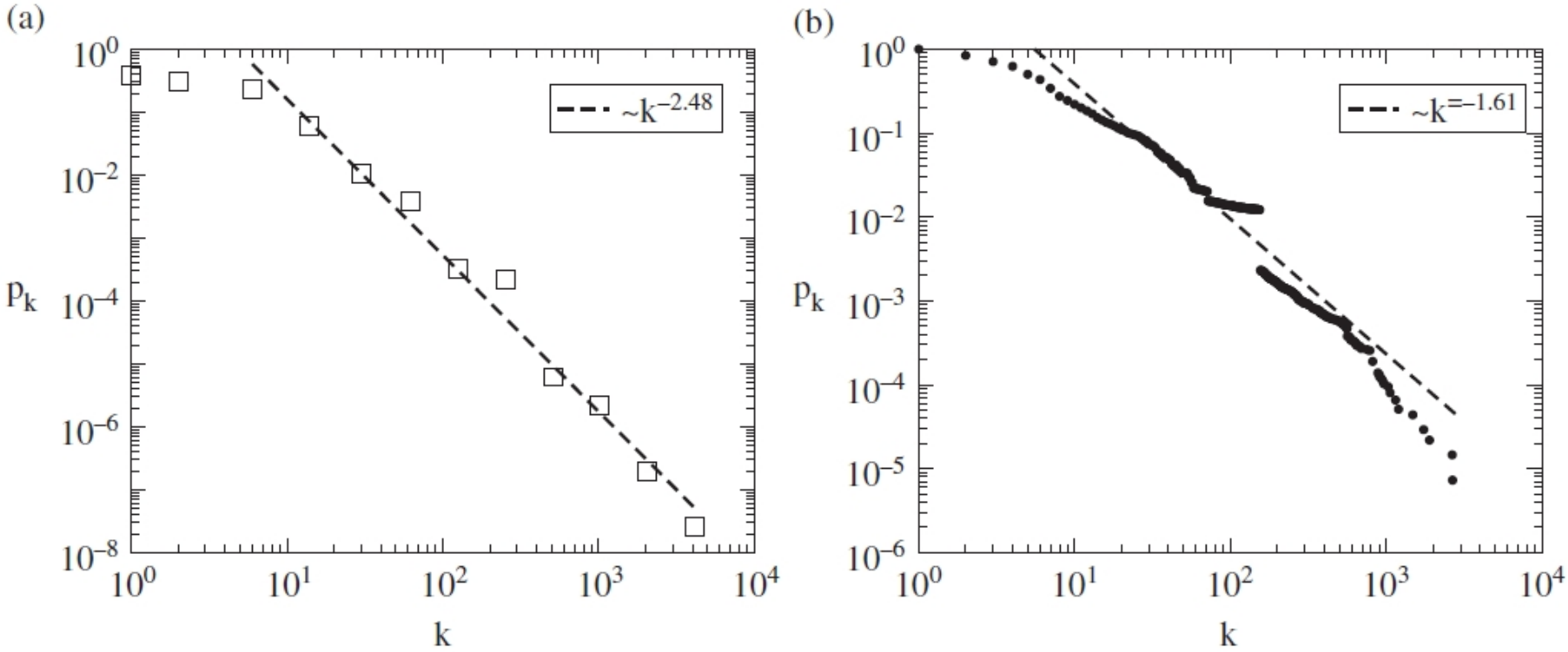

Figure 1: Out-degree distribution of Notre Dame WWW network. We report in (a) the histogram constructed by using a logarithmic binning, and in (b) the cumulative distribution.

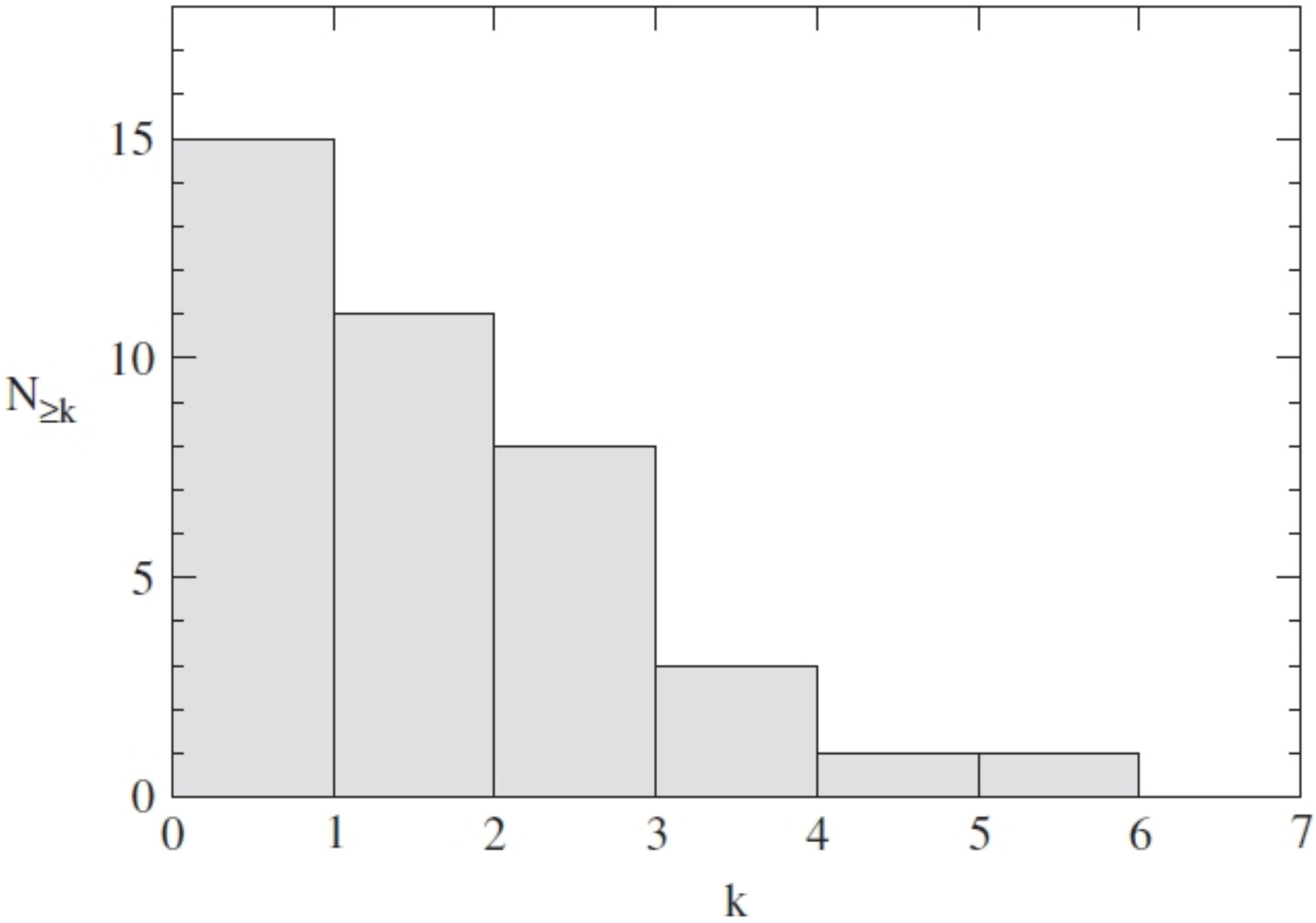

Figure 2: Consider the network of Florentine families.

Network	N	$\langle k \rangle$	γ	γ_{out}	γ_{in}
Notre Dame WWW	325729	4.51	–	2.06	1.91
Stanford WWW	281903	8.20	–	2.28	2.03
Berkley-Stanford WWW	685230	11.2	–	2.09	2.00
Google WWW	875713	5.83	–	3.63	2.57
AltaVista WWW from Ref. [59]	$2 \cdot 10^8$	7.5	–	2.72	2.1
Internet AS	11174	4.19	2.08	–	–
Internet routers	190914	6.36	2.54	–	–
Movie actor collaboration	225226	73.71	2.24	–	–
Cond-mat coauthorship	16726	5.69	3.57	–	–
Medline coauthorship	1520252	15.5	2.91	–	–
Sexual contacts from Ref.[208]	2810		2.6	–	–
Metabolic interactions from Ref.[166]	778	7.4	2.2	–	–
Protein interactions from Ref.[165]	1870	2.39	2.4	–	–

Figure 3: Power-law exponents characterising the degree distribution of real-world scale-free networks.

Computational Exercise

1. Read and compare your thoughts with a colleague. After let us make some discussion together of the following paper:
   - Perc Matjaz. The Matthew effect in empirical data *J. R. Soc. Interface* http://doi.org/10.1098/rsif.2014.0378.
2. Do the graph of Figure 2.
3. Obtain the γ for this network.
4. Explain to a colleague the meaning of this graph.
5. Go to Web of Science and explore some reports for the most important papers and authors in your area.
6. Study the h-index.
7. Study the Impact Factor and its relation to Qualis (Capes).
8. Study the current criteria to CNPq-PQ scholarship. Make some comparison with Network Science.