# ALGEBRAIC STRUCTURES RELATED WITH RATIONAL QUASI SEMI METRICS

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**Resumo:** We study in our work the rational quasi semi metrics. They can be understood as rational non-negative $n \times n$-matrices $A = (\alpha_{pq})$ with zero diagonal, such that

$$\alpha_{ij} + \alpha_{jk} \geqslant \alpha_{ik} \tag{0.6}$$

holds for all $i$, $j$ and $k$. If we assume that the entries $\alpha_{ij}$ are integers we obtain the definition of a non-negative exponent matrix. Exponent matrices are crucial ingredients of tiled orders and also related to semi maximal rings (see [1] for details). One of the interesting features of the non-negative exponent matrices is that they form a max-plus algebra. Another important remark is that (0.6) together with inequalities of non-negativity define a polyhedral cone.

The automorphisms of the next algebraic structures were described in [2]:

1. Max-plus algebra of (integer) non-negative exponent matrices;

2. Semigroups of (integer) non-negative exponent matrices with respect to addition and entry-wise maximum.

We describe the following algebraic structures:

1. Max-indecomposable (integer) exponent matrices;

2. Polyhedral cone of rational quasi semi metrics with respect, with binary operation sum, or entry-wise maximum; Its additive and max-automorphisms;

3. Group of max-automorphisms of the coordinate faces of the polyhedral cone of rational quasi semi metrics.

4. Group of combinatorial automorphisms of the polyhedral cone of rational quasi semi metrics [3];

This is a collaborated work with M. Dokuchaev and A. Mandel.

# Referências

[1] A. Zavadski and V. Kirichenko, *Torsion free modules over prime rings*, *Zap. Sci. Semin. LOMI USSR Akad. Sci.*, Vol. **57**, No 4, pp. 100-116, 1976.

[2] M. Dokuchaev, V. Kirichenko, G. Kudryavtseva and M. Plakhotnyk, *The max-plus algebra of exponent matrices of tiled orders*, *J. Algebra*, Vol. **490**, pp. 1–20, 2017.

[3] M. Dokuchaev, A. Mandel and M. Plakhotnyk, *The cone of Quase-semimetrics and exponent matrices of tiled orders*, *ArXiv*, eprint: 2005.05492, math.CO, may. 2020.

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