A GENETIC ALGORITHM FOR OPTIMIZATION OF GEOMETRICALLY NONLINEAR TRUSS STRUCTURES

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Abstract. Domes are structures largely used in arenas, exhibit pavilions, theaters, terminals, hangars, convention centers, gymnasiums, etc. The structural configuration of such structures allows for a completely column-free inner space. There are several possibilities of dome geometries frequently depending also on architectural considerations. Domes are often modeled as truss structures and a non-linear analysis is mandatory since stability constraints must be satisfied. The problem of choosing the sizes of the bars in order to minimize the weight of the structure while satisfying stress, displacement, stability, and other applicable constraints is complicated by the requirement of considering the non-linear structural behavior. The problem is further complicated if the bars are to be chosen from a discrete set of commercially available sizes, which is often the case. Genetic Algorithms, inspired by Darwin’s theory of evolution by natural selection, are powerful and versatile tools in difficult search and optimization problems. In this paper a genetic algorithm is proposed to find the optimum discrete values of the cross-sectional areas of the bars that minimize the weight of dome structures presenting geometrically non-linear behavior. Several computational experiments are discussed involving 30-, 52- and 120-bar dome structures as a test-bed.

Keywords: Optimization, Genetic Algorithm, Nonlinear Analysis
1. INTRODUCTION

Domes are economical structures largely used to cover arenas, exhibit pavilions, theaters, terminals, hangars, convention centers, gymnasiums, swimming pools, etc. The configuration of those structures allows for a completely free inner space without any interior column and they are considered elegant with their aesthetic appearance. There are several possible geometries for the domes depending frequently on the architectonic effect and the inventive ability of the architect. There are many types of latticed domes such as Schwedler, geodesic and lamella domes (Carbas and Saka (2009)). A preliminary engineering analysis is usually made of a few conventional configurations, pre-defined by the architect, and the engineer only chooses the sizes of the bars to satisfy the applicable codes, considering economic aspects. It is possible to consider the structure as a three-dimensional truss and the bars presenting joints non-rigidly interconnected. The normal and buckling stresses arising from the axial forces in the bars, and the displacements at the nodes, are the values that affect the sizes of the members and the final cost of the structure. For this kind of structure it is interesting to carry out a non-linear analysis to obtain the axial forces and displacements.

To consider a more realistic and adequate structural behavior of domes it had better to adopt the joints rigidly interconnected and the structure is analyzed as a space frame. Comparing the optimization using a linear analysis against a non-linear analysis it is possible, sometimes, to reach lighter weight considering the non-linear case. Also, a non-linear analysis requires the interaction between axial forces and bending moments in members with a high slenderness coefficient in order to check the stability of the structure Saka and Ulker (1991); Ebenau et al. (2005); Kameshki and Saka (2007). If the structure demands the non-linear analysis, such as the domes, the designer needs to adopt an structural optimization considering this behavior to define the optimal design. On the other, even if considering the domes as a three dimensional truss it important to perform a nonlinear analysis.

Among several references in this subject some of them are reported in this paper. In Saka and Kameshki (1997) an algorithm is presented for the optimum design of three dimensional rigidly jointed frames which takes into account the nonlinear response due to the effect of axial forces in members. The stability functions for three-dimensional beam-columns are used to obtain the nonlinear response of the frame. The problem of maximization of the critical load or limit point of instability of shallow space trusses of constant volume is presented in Kamat et al. (1984). In Pyrz (1990) a discrete optimization of trusses considering stability constraints is discussed presenting examples of shallow truss structures when snap-through can occur. Benchmark case studies in optimization of geometrically nonlinear structures can be found in Suleman and Sedaghati (2005) where a structural optimization algorithm is developed for truss and beam structures undergoing large deflections against instability.

In Saka (2007) an algorithm takes into account the nonlinear response of the dome structure due to effect of axial forces on the flexural stiffness of members and the optimum solution of the design problem is obtained using a coupled genetic algorithm.

A technique for the optimization of stability-constrained geometrically nonlinear shallow trusses with snap-trough behavior is demonstrated using the arc length method and a strain energy density approach with a discrete formulation in Hrinda and Nguyen (2008). In Degertekin et al. (2008) algorithms are presented for the optimum design of geometrically nonlinear steel space frames using tabu search and genetic algorithm.

In this paper, a GA is proposed to minimize the weight of dome structures as three dimensional trusses considering both geometrically linear and nonlinear analysis. Discrete and continuous design variables are considered corresponding to the sizing of the cross-sectional areas of the bars. To solve the nonlinear equilibrium equation of the structure the iterative
Newton-Raphson’s Method is adopted.

This paper is organized as follows. The structural optimization problem is presented in Section 2. The geometrically nonlinear approach is summarized in Section 3. The genetic algorithm and the constraint handling technique are discussed in Section 4. Numerical experiments are presented in Section 5. Finally, conclusions are presented in Section 6.

2. THE STRUCTURAL OPTIMIZATION PROBLEM

For a given objective function \( f(x) \), where \( x \in \mathbb{R}^n \) is the vector of design/decision variables, a standard structural sizing optimization problem reads: Find the set of areas \( x = \{ A_1, A_2, \ldots, A_N \} \) which minimizes the volume of the structure

\[
f(x) = \sum_{i=1}^{N} A_i l_i,
\]

where \( l_i \) is the length of the \( i \)-th bar of the truss and \( N \) is the number of bars. When shape design variables are considered the values of \( l_i \) change and the weight depends not only on the values of \( A_i \), but also on the joint coordinates of the structure.

The problem is usually subject to inequality constraints \( g_p(x) \geq 0 \), \( p = 1, 2, \ldots, \bar{p} \) and sometimes equality constraints \( h_q(x) = 0 \), \( q = 1, 2, \ldots, \bar{q} \). Also, the variables are usually subject to bounds \( x^L_i \leq x_i \leq x^U_i \) but this type of constraint is trivially enforced in a GA and does not require further consideration here. The most common constraints are normal stress constraints:

\[
\frac{\sigma_i}{\sigma_{max}} - 1 \leq 0, \quad i = 1, 2, \ldots, p_{\sigma}
\]

where \( \sigma_i \) is the normal stress at the \( i \)-th member and \( \sigma_{max} \) is the maximum allowable stress.

Displacements constraints can also be considered:

\[
\frac{|d_j|}{d_{max}} - 1 \leq 0, \quad k = 1, 2, \ldots, p_{d}
\]

where \( d_j \) is the displacement at the \( j \)-th global degree of freedom, \( d_{max} \) is the maximum allowable displacement, and \( p_{\sigma} + p_{d} = \bar{p} \). Additional constraints such as a minimum natural vibration frequency or more realistic buckling stress limits can also be included.

3. GEOMETRICALLY NONLINEAR APPROACH

Although the material of the structures discussed in this paper present a linear elastic behavior, geometrical non-linearity needs to be considered in the analysis. In order to provide an exact structural analysis, the equilibrium equation in each joint of the structure must be written considering the final geometry of the structure. In these equations nonlinear terms involving strain and displacement must be considered and the overall equilibrium equation can be written as:

\[
[K_T]\{u\} = \{P^*\}
\]

where

\[
[K_T] = [K_E] + [K_G]
\]

and \([K_T]\) is called overall tangent stiffness matrix of the structure, \([K_E]\) is known as the overall linear elastic stiffness matrix and \([K_G]\) is the geometric stiffness matrix. The matrix \([K_G]\)
depends on the elastic and geometric stiffness matrix and \( \{P^*\} \) is the vector of unbalanced load. To solve the equation (5) an iterative scheme is required and here the Newton-Raphson’s Method is adopted. Newton’s Method is summarized as follow:

1. Perform the linear analysis of the structure and obtain the displacements for the first load step;
2. Update the joint coordinates of the structure considering the displacements obtained in the previous step;
3. Evaluate the internal member actions;
4. Evaluate, at each node, the resultant of the internal member actions in the global axes;
5. Evaluate the unbalanced load vector that is the difference between the member actions obtained in the previous step and the load applied at the structure in this load step;
6. Assemble the stiffness matrix and check its determinant. The loss of stability is identified if the determinant presents a negative value. If it is positive the structure is analyzed considering the unbalanced load vector and new incremental displacements are obtained;
7. Update the node coordinates;
8. Repeat steps 3 to 7 until the unbalanced vector satisfies the error tolerance.

It is important to note that in the nonlinear procedure used in this paper when the loss of stability occurs the current displacements and internal actions in the bars are amplified by a factor of 100. This candidate solution is strongly penalized by the penalty scheme and consequently has a low position in the rank of the population.

4. THE GA AND THE PENALTY SCHEME

A rank-based selection scheme is adopted in the binary-coded GA used here where the selection scheme operates on the current population sorted according to the values of the fitness function where better solutions have higher rank. The candidate solutions presenting lower fitness values will have a higher rank considering weight minimization problems discussed in this paper. A form of elitism is used where the best individual is always copied into the next generation along with one copy where one randomly chosen bit has been changed. The recombination of the genetic material of the selected “parent” chromosomes uses the standard uniform crossover operator applied with probability equal to \( p_{\text{cross}} = 0.8 \). A mutation operator is introduced with a mutation rate \( p_m = 0.03 \) applied to each bit in the offspring chromosomes. The whole process is repeated for a given number of generations or until certain stopping criteria are met. A pseudo-code of the binary GA used here is displayed in the Figure 1.

The adaptive penalty method introduced by Barbosa and Lemonge (2002), and applied to structural optimization problems in Lemonge and Barbosa (2004), will be applied here to enforce all the mechanical constraints considered in the numerical experiments (stresses and displacements). Defining the amount of violation of the \( j \)-th constraint by the candidate solution \( x \) as

\[
v_j(x) = \begin{cases} |h_j(x)|, & \text{for an equality constraint}, \\
\max\{0, -g_j(x)\}, & \text{otherwise}
\end{cases}
\]
Algorithm generational GA
Initialize the population P
Evaluate individuals in population P using a linear or nonlinear analysis
repeat
  Copy elite to P’
  repeat
    Select 2 or more individuals in P
    Apply a recombination operator with probability $p_c$
    Apply mutation operator with rate $p_m$
    Insert new individuals in P’
  until population P’ complete
  Evaluate individuals in population P’ using a linear or nonlinear analysis
  $P ← P’$
until stopping criteria are met

*Figure 1:* A standard binary encoded genetic algorithm.

it is common to design penalty functions that grow with the vector of violations $v(x) ∈ R^M$ where $M = \bar{p} + \bar{q}$ is the number of constraints to be penalized. The fitness function is defined as Barbosa and Lemonge (2002)

$$F(x) = \begin{cases} f(x), & \text{if } x \text{ is feasible}, \\ \bar{f}(x) + \sum_{j=1}^{M} k_j v_j(x) & \text{otherwise} \end{cases}$$

(6)

where

$$\bar{f}(x) = \begin{cases} f(x), & \text{if } f(x) > \langle f(x) \rangle, \\ \langle f(x) \rangle & \text{otherwise} \end{cases}$$

(7)

and $\langle f(x) \rangle$ is the average of the objective function values in the current population.

The penalty parameter is defined at each generation by:

$$k_j = |\langle f(x) \rangle| \frac{\langle v_j(x) \rangle}{\sum_{l=1}^{M} |\langle v_l(x) \rangle|^2}$$

(8)

and $\langle v_l(x) \rangle$ is the violation of the $l$-th constraint averaged over the current population.

### 5. NUMERICAL EXPERIMENTS

Three experiments are discussed in this section i.e. 30-bar, 52-bar and 120-bar truss dome. For each one of them the discrete and continuous as well as the linear and nonlinear cases are analyzed. For the discrete cases the cross-sectional areas of the sizing design variables are to be chosen from the Table containing 64 tubular sections shown in Table 1. For the continuous case the bounds of the sizing design variables are $1.0 \, cm^2 ≤ A_i ≤ 220 \, cm^2$. The density of the material is $7.86 \times 10^{-5} kN/cm^3$ and Young’s modulus is equal to 21000 kN/cm². The maximum normal stress is limited by $σ_{max} = 30 \, kN/cm^2$ in tension and compression. The maximum displacement in any direction is set equal to 4 cm. Six independent runs were performed considering 20 individuals in the population. For the discrete cases the populations was evolved for 40 generations, except in the third experiment when 60 was set as the maximum
number of generations. For the continuous cases the maximum number of generations was set equal to 140 for the first and the third experiments and 80 for the second experiment. The number of bits was set equal to 15 for each design variable. When the nonlinear analysis is performed, eight load steps and eight iterations, per load step, were adopted in the iterative Newton-Raphson’s Method.

<table>
<thead>
<tr>
<th>Section cm²</th>
<th>Section cm²</th>
<th>Section cm²</th>
<th>Section cm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 1.238</td>
<td>17 9.892</td>
<td>33 29.940</td>
<td>49 87.510</td>
</tr>
<tr>
<td>2 1.583</td>
<td>18 10.690</td>
<td>34 30.010</td>
<td>50 98.000</td>
</tr>
<tr>
<td>3 1.799</td>
<td>19 11.170</td>
<td>35 34.790</td>
<td>51 112.400</td>
</tr>
<tr>
<td>4 2.291</td>
<td>20 12.260</td>
<td>36 34.820</td>
<td>52 129.500</td>
</tr>
<tr>
<td>5 2.919</td>
<td>21 12.520</td>
<td>37 39.640</td>
<td>53 126.700</td>
</tr>
<tr>
<td>6 3.345</td>
<td>22 15.170</td>
<td>38 40.400</td>
<td>54 138.800</td>
</tr>
<tr>
<td>7 3.510</td>
<td>23 15.400</td>
<td>39 46.030</td>
<td>55 141.100</td>
</tr>
<tr>
<td>8 4.029</td>
<td>24 15.520</td>
<td>40 49.270</td>
<td>56 148.700</td>
</tr>
<tr>
<td>9 4.205</td>
<td>25 17.070</td>
<td>41 57.270</td>
<td>57 155.500</td>
</tr>
<tr>
<td>10 4.564</td>
<td>26 19.120</td>
<td>42 58.900</td>
<td>58 167.100</td>
</tr>
<tr>
<td>11 5.760</td>
<td>27 19.130</td>
<td>43 65.190</td>
<td>59 167.800</td>
</tr>
<tr>
<td>12 6.465</td>
<td>28 22.720</td>
<td>44 68.500</td>
<td>60 169.800</td>
</tr>
<tr>
<td>13 7.100</td>
<td>29 25.160</td>
<td>45 69.130</td>
<td>61 196.200</td>
</tr>
<tr>
<td>14 7.349</td>
<td>30 25.220</td>
<td>46 73.060</td>
<td>62 213.800</td>
</tr>
<tr>
<td>15 7.591</td>
<td>31 26.320</td>
<td>47 78.040</td>
<td>63 217.300</td>
</tr>
<tr>
<td>16 8.636</td>
<td>32 29.170</td>
<td>48 87.360</td>
<td>64 221.800</td>
</tr>
</tbody>
</table>

5.1 The 30-bar dome truss

The 30-bar three-dimensional dome depicted in Figure 4 is subjected to a weight minimization considering a load of 20.0 kN applied in the vertical downward direction at the node 1. The baseline coordinates of the nodes are displayed in Table 2 and the bars are linked in four groups: i) group 1: 1 to 6; ii) group 2: 7 to 12; iii) group 3: 13-18; group 4: 19 to 30.

<table>
<thead>
<tr>
<th>node</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0</td>
<td>0.0</td>
<td>218.216</td>
</tr>
<tr>
<td>3</td>
<td>914.4</td>
<td>0.0</td>
<td>164.241</td>
</tr>
<tr>
<td>4</td>
<td>457.2</td>
<td>791.984</td>
<td>164.241</td>
</tr>
<tr>
<td>11</td>
<td>1828.8</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>12</td>
<td>1371.6</td>
<td>791.894</td>
<td>55.141</td>
</tr>
<tr>
<td>13</td>
<td>914.4</td>
<td>1583.787</td>
<td>0.0</td>
</tr>
<tr>
<td>14</td>
<td>0.0</td>
<td>1583.787</td>
<td>55.141</td>
</tr>
</tbody>
</table>

5.2 The 52-bar dome truss

The 52-bar three-dimensional dome depicted in Figure 3 is subject to weight minimization considering vertical loads of 150.0 kN of magnitude applied in the downward direction at the
Figure 2: The 30-bar truss dome.
nodes 6 to 13. There are 8 member groups as indicated in the Figure 3.

![Figure 3: The 52-bar truss dome.](image)

### 5.3 The 120-bar dome truss

The dome depicted in Figure 2 is a 120-bar three-dimensional truss previously discussed in the references Saka and Ulker (1991); Ebenau et al. (2005); Capriles et al. (2007). The dome is subject to a downward vertical equipment loading of 600 kN at its crown (node 1).

### 5.4 Results

Tables 3 and 4 present the best solution found for the discrete and continuous cases respectively where “dv” means design variable. All the solutions shown in these Tables are rigorously feasible where the stress and displacements are into the allowable required limits for these constraints. For both cases the 30-bar truss dome presented a reasonable difference between the optimization considering linear and nonlinear analysis. A difference of 7.79 % (19.802 kN against 21.344 kN), between these solutions was encountered where the nonlinear analysis leads to a heavier final weight (21.344 kN). In the continuous case it occurred again and the difference was 7.29 % (19.814 kN against 21.259 kN), in favor of the solution carried out using the linear analysis. It is interesting to show the importance of performing both linear and nonlinear analysis, particularly, in these types of structures.

On the other hand, the linear and nonlinear analysis of the 52- and 120-bar truss dome, for
Figure 4: The 120-bar truss dome.
both discrete and continuous case, present a slight difference. For the discrete case, considering
the 52-bar truss dome, the difference between the final weights was 1.06 % (12.303 kN against
12.433 kN), where the linear analysis reached the lighter weight. For the continuous case, the
difference was 1.45 % and the linear analysis reached the lighter weight. Again, it is important
to observe the necessity to be careful in deciding the type of analysis to be adopted. The designer
have to be assured with respect to each analysis to be conducted for the design. If the linear
analysis is selected ignoring the nonlinear analysis it can be dangerous.

Finally, for the 120-bar truss dome the final weight considering the discrete case reached
a difference equal to 1.60 % (38.927 kN against 39.552 kN). For the continuous case the dif-
ference was 2.40 % (38.140 kN against 39.057 kN) showing the importance of carry out the
nonlinear analysis in the optimization process since lighter weights were found using linear
analysis. Again, the designer must be observe the differences between these analysis. Figures
from 5 to 10 present the evolution of best solution each experiment.

An important detail must be observed. Although, it is possible to reach a non significant dif-
ference in the final weights using linear and nonlinear analysis, for example, in the discrete case
of the 120-bar the final weights are 38.927 kN (linear analysis) against 39.552 kN (non-linear)
analysis with a slight difference of 1.6 % but the final cross-sectional areas are significantly
distinct i.e. 19.120 cm$^2$ versus 22.720 cm$^2$ for $A_1$ with a difference of 18.83 %; 7.10 cm$^2$ ver-
sus 9.892 cm$^2$ for $A_2$ with a difference of 39.32 %; 8.636 cm$^2$ versus 6.465 cm$^2$ for $A_3$ with
a difference of 33.58 %; 19.120 cm$^2$ versus 13.070 cm$^2$ for $A_4$ with a difference of 46.29 %;
5.76 cm$^2$ versus 3.345 cm$^2$ for $A_5$ with a difference of 72.19 %. For this experiment the final
solutions for the cross-sectional $A_7$ present the same value equal to 1.583 cm$^2$. This fact can be
observed in others results presented in Tables Tables 3 and 4. The performance of GA for each
experiment is provided in Table 5 where the six first part of the table corresponds to the discrete
case and the other one to the continuous case.

### Table 3: Final weights in kN for the discrete cases.

<table>
<thead>
<tr>
<th>dv</th>
<th>30-bar</th>
<th>52-bar</th>
<th>120-bar</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LIN</td>
<td>NL</td>
<td>LIN</td>
</tr>
<tr>
<td>$A_1$</td>
<td>22.720</td>
<td>25.220</td>
<td>1.238</td>
</tr>
<tr>
<td>$A_2$</td>
<td>12.260</td>
<td>15.170</td>
<td>1.238</td>
</tr>
<tr>
<td>$A_4$</td>
<td>1.583</td>
<td>1.238</td>
<td>1.238</td>
</tr>
<tr>
<td>$A_5$</td>
<td>–</td>
<td>–</td>
<td>3.510</td>
</tr>
<tr>
<td>$A_6$</td>
<td>–</td>
<td>–</td>
<td>4.029</td>
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<tr>
<td>$A_7$</td>
<td>–</td>
<td>–</td>
<td>6.760</td>
</tr>
<tr>
<td>$A_8$</td>
<td>–</td>
<td>–</td>
<td>4.029</td>
</tr>
</tbody>
</table>
Table 4: Final weights in kN for the continuous cases.

<table>
<thead>
<tr>
<th>dv</th>
<th>30-bar</th>
<th>52-bar</th>
<th>120-bar</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>22.761</td>
<td>22.193</td>
<td>1.394</td>
</tr>
<tr>
<td>A2</td>
<td>17.408</td>
<td>19.379</td>
<td>1.488</td>
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<tr>
<td>A3</td>
<td>3.667</td>
<td>5.585</td>
<td>1.708</td>
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<tr>
<td>A4</td>
<td>1.000</td>
<td>1.000</td>
<td>1.454</td>
</tr>
<tr>
<td>A5</td>
<td></td>
<td>4.963</td>
<td>5.130</td>
</tr>
<tr>
<td>A6</td>
<td></td>
<td>6.447</td>
<td>5.999</td>
</tr>
<tr>
<td>A7</td>
<td></td>
<td>2.818</td>
<td>3.386</td>
</tr>
<tr>
<td>A8</td>
<td></td>
<td>2.410</td>
<td>1.067</td>
</tr>
<tr>
<td>W</td>
<td>19.814</td>
<td>21.259</td>
<td>10.944</td>
</tr>
</tbody>
</table>

Table 5: Performance of the GA for each experiment.

<table>
<thead>
<tr>
<th>30-bar</th>
<th>best</th>
<th>average</th>
<th>median</th>
<th>std. dev.</th>
<th>worst</th>
</tr>
</thead>
<tbody>
<tr>
<td>NLI</td>
<td>21.344</td>
<td>22.576</td>
<td>22.704</td>
<td>1.258E+00</td>
<td>24.659</td>
</tr>
<tr>
<td>52-bar</td>
<td>LIN</td>
<td>12.303</td>
<td>15.736</td>
<td>15.122</td>
<td>1.842E+00</td>
</tr>
<tr>
<td>NLI</td>
<td>12.433</td>
<td>16.430</td>
<td>16.470</td>
<td>2.432E+00</td>
<td>19.970</td>
</tr>
<tr>
<td>120-bar</td>
<td>LIN</td>
<td>38.927</td>
<td>41.251</td>
<td>41.167</td>
<td>2.052E+00</td>
</tr>
<tr>
<td>NLI</td>
<td>39.552</td>
<td>41.564</td>
<td>41.670</td>
<td>1.505E+00</td>
<td>44.194</td>
</tr>
</tbody>
</table>

Figure 5: Evolution of the best discrete solution of the 30-bar dome truss.
Figure 6: Evolution of the best continuous solution of the 30-bar dome truss.

Figure 7: Evolution of the best discrete solution of the 52-bar dome truss.
Figure 8: Evolution of the best continuous solution of the 52-bar dome truss.

Figure 9: Evolution of the best discrete solution of the 120-bar dome truss.
6. CONCLUSIONS

In this paper, a GA is used to minimize the weight of dome structures as three dimensional trusses considering geometrically linear as well as nonlinear analysis. Discrete and continuous design variables were considered corresponding to the sizing of the cross-sectional areas of the bars. To solve the nonlinear equilibrium equation of the structure the iterative Newton-Raphson’s Method was adopted.

One can observe from the results of the analysis the importance of carry out the linear and nonlinear procedures since these analysis can lead to different final weights, manly, the final values of the cross-sectional areas. The designer have to be attempt in order to choice the adequate analysis to be conducted.

The GA used in this paper has been improved in other aspects to solve more complex optimization problems considering nonlinear and snap-through analysis as well as problems of maximization the critical load or limit point of instability.

Also, several aspects of the iterative methods have to be considered in order to accelerate the search of the optimum solutions.

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